

online riordanica

encyclopedia

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Preface

Triangular arrays and general Riordan arrays are studied in J. O. Shallit (Univ. of Waterloo) 1980 paper "A triangle for the Bell numbers" and A. Nkwanta (Morgan State Univ.) articles, cited below. For the History, we recall that these are the starting points of the "Online Encyclopedia Riordanica (OERIOR)" with the purpose to encourage research on topics related to Riordan arrays/Riordan group, to provide assistance in the preparation of a thesis, to stimulate graduate students and researchers who want to get more insight on a specific topic, to provide References and Citations for new Publications.

In 2014, OERIOR included only about hundred Publications. I sent this material to A. Nkwanta and G-S. Cheon (Sungkyunkwan Univ.), with a kind request to express their opinion on the project. Their prompt and enthusiastic response encouraged me to pursue the "Online Encyclopedia Riordanica (OERIOR)".

1.- Introduction

Oerrior is articulated in 3 parts: Database, Glossary, Bibliography. Database is a survey of articles relevant to Oerrior, Glossary is a survey of labelled Directories and Bibliography is a list of recommended readings. Each part is

accessible by clicking on the corresponding Link (see below). Citations are written in black if a free copy is available and/or an Open Access policy is applicable and in red if only an Abstract is available due to Purchase requests. The articles in the Database are ordered by the authors family names and date of publication; when papers by the same authors appear the same year, we also use letters a, b, c, etc...after the year, as in the following examples:

Azarian2012a, Fibonacci identities as binomial sums, Int. J. Contemp. Math. Sci. Vol. 7, 2012, no. 38, 1871-1876, [gen>](#)

Azarian2012b, Fibonacci identities as binomial sums II, Int. J. Contemp. Math. Sci. Vol. 7, 2012, no. 42, 2053- [>gen>](#)

Azarian2012c, Identities involving Lucas or Fibonacci and Lucas numbers as binomial sums, Int. J. Contemp. Math. Sci. Vol. 7, 2012, no. 45, 2221-2227, [gen>](#)

Cheon G-S.2003, A note on the Bernoulli and Euler polynomials, Appl. Math. Letters Vol. 16, Issue 3, Apr 2003, 365–368, [gen>](#)

Cheon G-S.El-Mikkawy2007, Generalized harmonic numbers identities and a related matrix representation, J. Korean Math. Soc. 2007 Vol. 44, No. 2, 487-498, [nat>](#)

Cheon G-S.El-Mikkawy2008, Generalized harmonic numbers with Riordan arrays, J. Number Theory Vol. 128, Issue 2, Feb 2008, 413–425, [jou>](#)

Cheon G-S.HwangRimSong2003, Matrices determined by a linear recurrence relation among entries, Linear Algebra Appl Vol. 373, Nov 2003, 89–99, [gen>](#)

Cheon G-S.Jin2011, Structural properties of Riordan matrices and extending the matrices, Linear Algebra Appl Vol. 435, Issue 8, Oct 2011, 2019–2032, [gen>](#)

- Cheon G-S.JinKimShapiro2009, Riordan group involutions and the q -sequence, Discrete Appl. Math. 157 (2009) 1696-1701, [gen](#)
- Cheon G-S.Kim2001, Stirling matrix via Pascal matrix, Linear Algebra Appl. Vol. 329, Issues 1–3, May 2001, 49–59, [gen>](#)
- Cheon G-S.Kim2002, Factorial Stirling matrix and related combinatorial sequences, Linear Algebra Appl. Vol. 357, Issues 1–3, Dec 2002, 247–258, [gen](#)
- Cheon G-S.Kim2008, Simple proofs of open problems about the structure of involutions in the Riordan group, Linear Algebra Appl. Vol. 428, Issue 4, Feb 2008, 930–940, [gen>](#)
- Cheon G-S.KimShapiro2008, Riordan group involutions, Linear Algebra Appl. Vol. 428, Issue 4, Feb 2008, 941–952, [gen>](#)
- Cheon G-S.KimShapiro2009, A generalization of Lucas polynomial sequence, Discrete Appl. Math. Vol. 157, Issue 5, Mar 2009, 920–927, [gen>](#)
- Cheon G-S.KimShapiro2012, Combinatorics of Riordan arrays with identical A and Z sequences, Discrete Math. Vol. 312, Issues 12–13, Jul 2012, 2040–2049, [gen >](#)
- Cheon G-S.YungLim2013, A q -analogue of the Riordan group, Linear Algebra Appl Vol. 439, Issue 12, Dec 2013, 4119–4129, [gen>](#)
- Nkwanta2003, A Riordan matrix approach to unifying a selected class of combinatorial arrays, Congr. Numer. 160 (2003), 33-45, [gen>](#)
- Nkwanta2008, Lattice Paths, Riordan Matrices and RNA Numbers, Congr. Numer. 01/2008, [gen>](#)

Nkwanta2009, Lattice path and RNA secondary structure predictions, 15th Conf. African American Researchers Math. Sci.-Rice Univ., Jun 23-26, 2009, [gen>](#)

Nkwanta2010, Riordan matrices and higher-dimensional lattice walks, J. of Statist. Plann. Inference Vol. 140, Issue 8, Aug 2010, 2321–2334, [<jou>](#)

NkwantaBarnes2012, Two Catalan-type Riordan arrays and their connections to the Chebyshev polynomials of the first kind, J. Integer Seq. Vol. 15 (2012), Article 12.3.3, [<jis>](#)

NkwantaKnox1999, A note on Riordan matrices, Thesis-Contemp. Math. Vol. 252. 1999, Howard University, Washington, DC 1997, [gen>](#)

NkwantaShapiro2005, Pell walks and Riordan matrices, Fibonacci Quart. 2005 (43,2): 170-180, [<fibqy>](#)

NkwantaTefera2013, Curious relations and identities involving the Catalan generating function and numbers, J. of Integer Seq. Vol. 16 (2013), Article 13.9.5, [jis>](#)

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The essential part of Oerrior is Glossary; this Section is a collection of labelled Directories where the labels are words appearing in the titles of the Publications listed in Database. These labels play the role of keywords. A title may define more than one label and terms in different labels may coincide. A Directory contains the articles with a particular keyword in the title. Directories are always displayed in the same way as, for instance, in the following Directory indexed by Meixner:

- Keywords are written in green letters,
- Keywords names are followed by the list of articles contained in the corresponding Directory,
- When the list is long, we divide it in categories such as, Meixner-Riordan arrays, < Meixner-

type and Meixner polynomials

Meixner

BozejkoDemni2010, Topics on Meixner families, Banach Center Publications, 2010 Vol. 89, 61-74, nat

Meixner-Riordan arrays BarryHennessey2010b, Meixner-type results for Riordan arrays and associated integer sequences, J. Integer Seq. Vol. 13 (2010), Article 10.9.4, jis Meixner-type

BarryHennessey2010b, Meixner-type results for Riordan arrays and associated integer sequences, J. Integer Seq. Vol. 13 (2010), Article 10.9.4, jis Meixner polynomials

Alvarez-NodarseMarcellan1995b, Difference equation for modifications of Meixner polynomials, J. Math. Anal. Appl. Vol. 194, Issue 1, Aug 1995, 250–258, jou

Bavinck, van Haeringen1994, Difference equations for generalized Meixner polynomials, J. Math. Anal. Appl. Vol. 184, Issue 3, Jun 1994, 453–463, jou

GriffithsSpano2011, Multiv. Jacobi and Laguerre polyn., infinite-dimens. extensions and their prob. connect. with multiv. Hahn and Meixner polynomials, Bernoulli 17 (3), 2011, 1095–1125, gen

KhanAkhlaq2012, A note on generating functions and summation formulas for Meixner polynomials of several variables, Demonstratio Math. Vol. XLV, No. 1, 2012, gen

Shibukawa2014, Multivariate Meixner, Charlier and Krawtchouk polynomials, arXiv (29 Apr 2014), aXv>

generating functions

KhanAkhlaq2012, A note on generating functions and summation formulas for Meixner polynomials of several variables, Demonstratio Math. Vol. XLV, No. 1, 2012, gen

Hahn

GriffithsSpano2011, Multiv. Jacobi and Laguerre polyn., infinite-dimens. extensions and their prob. connect. with multiv. Hahn and Meixner polynomials, Bernoulli 17 (3), 2011, 1095–1125, gen

integer sequences

BarryHennessey2010b, Meixner-type results for Riordan arrays and associated integer sequences, J. Integer Seq. Vol. 13 (2010), Article 10.9.4, jis

Jacobi (see also elliptic)

GriffithsSpano2011, Multiv. Jacobi and Laguerre polyn., infinite-dimens. extensions and their prob. connect. with multiv. Hahn and Meixner polynomials, Bernoulli 17 (3), 2011, 1095–1125, gen

Shibukawa2014, Multivariate Meixner, Charlier and Krawtchouk polynomials, arXiv (29 Apr 2014), aXv

Laguerre

GriffithsSpano2011, Multiv. Jacobi and Laguerre polyn., infinite-dimens. extensions and their prob. connect. with multiv. Hahn and Meixner polynomials, Bernoulli <17 (3), 2011, 1095–1125, gen>

Shibukawa2014, Multivariate Meixner, Charlier and Krawtchouk polynomials, arXiv (29 Apr 2014), aXv>

moments BrycWesolowski2004, Conditional moments of q-Meixner processes, arXiv (13 Dec 2004), aXv>

process

BrycWesolowski2004, Conditional moments of q-Meixner processes, arXiv (13 Dec 2004), aXv>

The above display is a so-called thematic map; more precisely we say that it is the Meixner Directory thematic map. Usually a thematic map is related to several keywords, in our case: generating functions, Hahn, integer sequences, Jacobi (see also elliptic), Krawtchouk, Laguerre,

moments and process. Displaying the additional thematic maps, we get 9 thematic maps which provide a more detailed panoramic view of the topic. To save space, we have not displayed these thematic maps. Thematic maps are used in conjunction with the applications mentioned above; they represent an important feature of OERIOR.

[Abel](#)

[Akiyama-Tanigawa](#)

[Al-Salam-Carlitz](#)

[Al Salam Chihara](#)

[Apery](#)

[Apostol](#)

[Apostol-Bernoulli](#)

[Apostol-Euler](#)

[Apostol-Genocchi](#)

[Appel](#)

[Array Type Polynomials](#)

[Askey Scheme](#)

[Askey Wilson Algebra](#)

[Askey-Wilsonbasis](#)

[Bell](#)

[Bell Partial Polynomials](#)

[Bernoulli](#)

[Bernstein](#)

[Bessel Big Q Analogues](#)

[Bessel](#)

[Binet Formula](#)

[binomial](#)

[Brownian Motion, Brownian Motion Q Analogue](#)

[Carlitz](#)

[Catalan](#)

[Cauchy](#)

[Central Coefficients](#)

[Central Factorial Numbers](#)

[Chan Chyan Srivastava](#)

[Charlier](#)
[Chebyshev \(Tschebyscheff\)](#)
[Chebyshev Boubaker](#)
[Coefficients Method](#)
[Cohen Macaulay Property](#)
[Combinatorial Theory](#)
[Comtet](#)
[Congruences](#)
[Connection Coefficients](#)
[Continued Fractions](#)
[Convolution](#)
[Cumulants](#)
[Daehee](#)
[Daehee](#)
[Denert Statistic](#)
[Darangements, Darangements 0 Analogues](#)
[Diophantine Equations](#)
[Dobinski](#)
[Dumont Foata](#)
[Ehrhart](#)
[Elliptic \(see Also Jacobi\)](#)
[Embedding Distributions, Structures](#)
[Entriger](#)
[Entropy](#)
[Erkus Srivastava](#)
[Euler](#)
[Euler Barnes](#)
[Euler Bernoulli](#)
[Euler Frobenius](#)
[Eulerian](#)
[Euler Seidel](#)
[Faber](#)
[Factorial Generalizations](#)
[Fibonacci](#)
[Fibonacci Lucas](#)
[Fibonomial Coefficients](#)
[Fine](#)

[Frobenius](#)
[Gandhi](#)
[Gauss \(see Also Hypergeometric\)](#)
[Gegenbauer \(see Also Ultraspherical\)](#)
[Gegenbauer Humbert](#)
[Generating Functions](#)
[Genocchi](#)
[Hahn](#)
[Hankel](#)
[Harmonic](#)
[Hermite](#)
[Hermite Big Q Polynomials](#)
[Hessenberg](#)
[Horadam](#)
[Humbert](#)
[Hypergeometric \(see Also Gauss\)](#)
[Identities, Inequalities](#)
[Incomplete Numbers, Generalized Numbers, Polynomials](#)

[Integer Sequences](#)
[Inverse \(reciprocal\) Numbers, Sums, Polynomials](#)
[inversion-techniques.pdf" target="">Inversion Techniques](#)
[Jacobi \(see Also Elliptic\)](#)
[Jacobi Big Q Polynomials](#)
[Jacobi Little Q Polynomials](#)
[Jacobi Stirling](#)
[Jacobsthal](#)
[Jacobsthal Lucas](#)
[Konhauser](#)
[Krawtchouk](#)
[Lacunary Series](#)
[Lagrange](#)
[Laguerre Little Q Polynomials](#)
[Laguerre](#)
[Lah](#)
[Lattice](#)
[Laurent](#)

[LDU Decomposition, Cholesky Factorization](#)

[Legendre](#)

[Legendre Stirling](#)

[Lehmer](#)

[Lehner](#)

[Lengyel](#)

[Linear Algebra Of Certain Matrices](#)

[Lucas](#)

[Lucas Bernoulli](#)

[Lucasian](#)

[Mahonian Pairs, Statistics](#)

[Meixner](#)

[Mellin](#)

[Manage Problem](#)

[Mixed Type Polynomials](#)

[Modular](#)

[Moments](#)

[Morgan Voyce](#)

[Motzkin](#)

[Narayana](#)

[Narumi](#)

[N Bonacci Numbers](#)

[Newton Series](#)

[Norlund](#)

[Norlund Bernoulli](#)

[Norlund Euler](#)

[Operational Calculus](#)

[Oresme](#)

[Orthogonal \(q \)polynomials](#)

[Partial Euler Product](#)

[Pascal](#)

[Paths](#)

[Patterns](#)

[Pell](#)

[Pell Equation, Pell Abel Equation](#)

[Pell Lucas](#)

[Permanents](#)
[Permutations](#)
[Perrin](#)
[Poisson Charlier](#)
[Poly Numbers, Poly Polynomials](#)
[Posets](#)
[Process](#)
[Production Matrices](#)
[Q Analogue Calculus](#)
[Racah Coefficients](#)
[Recurrence Relations](#)
[Renewal Array, Process](#)
[Riemann \(see Also Z Function\)](#)
[Riordan Group, Q Analogue](#)
[RNA Secondary Structures, Numbers](#)
[Rodrigues](#)
[Salié](#)
[Schröder](#)

[Schubert](#)
[Schur](#)
[Seidel Arnoldi](#)
[Selberg](#)
[Sheffer Group](#)
[Sheffer Polynomial Sequences](#)
[Sheffer Type](#)
[Sobolev](#)
[Somos 4 Sequences](#)
[Springer](#)
[Srivastava](#)
[Srivastava Pinter Addition Theorems](#)
[Stern Brocot Sequence](#)
[Stieltjes](#)
[Stirling](#)

Stirling generalized numbers group
[Stochastic Processes](#)

[Succession Rules](#)

[Sulanke](#)

[Tangent Numbers, Tanh Numbers](#)

[Tetranacci](#)

[Toda Chain](#)

[Toeplitz](#)

[Toeplitz Plus Hankel](#)

[Touchard](#)

[Transforms](#)

[Tribonacci](#)

[Tribonacci Lucas](#)

[Ultraspherical \(see Also Gegenbauer\)](#)

[Umbral Calculus](#)

[Van Der Laan](#)

[Vandermonde](#)

[Vieta Jacobsthal Lucas, Vieta Pell Lucas Polynomials](#)

[Vieta, Vieta Jacobsthal, Vieta Pell Polynomials](#)

[Weierstrass](#)

[Wiener Chaos](#)

[Wythoff Number, Pair](#)

[Zernike](#)

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Database (to see the publications listed in the Database, CTRL and click here [Database](#)).

Glossary-Keywords (to see the details of Glossary-Keyword, CTRL and click here [Glossary-Keywords](#)).

Glossary (to see the details of Glossary, CTRL and click here [<Contents](#)).

Bibliography (to see the items in the Bibliography, CTRL and click here [Bibliography](#)).

Conclusion

Any item in OERIOR can be read on-line (CTRL and one: jis>, aXv>, gen>, jou>, nat>, fibqy (acronyms of Journal Integer Sequences, aXv, General, Journal, Fibquarterly, National). This original feature of OERIOR gives immediate access to desired information.

There are 1959 entries in Database, 192 in Glossary and 79 in Bibliography . These numbers grow as new items are discovered in the literature due to reader ' contributions. Readers are welcome to send via email suggestions for further additions.

By inspection we can see that in Database only few items have in the title the keywords "Riordan arrays/group"; the others are included in OERIOR because they also belong to a Directory related (àCTRL and click here Contents) to a Directory indexed by Riordan arrays/group.

OERIOR is open/free and may be copied for personal reading. We kindly ask users to publicize OERIOR by including in their publications the Reference "G. DellaRiccia, Online Encyclopedia Riordanica", the Citation "Online Encyclopedia Riordanica (Oerior)" and the Link

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Giacomo Della Riccia (May 2017)