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BILATERAL GENERATING RELATIONS FOR A FUNCTION DEFINED BY GENERALIZED RODRIGUES FORMULA

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Several authors, including Srivastava and Panda (1975) have defined polynomial sets by means of Rodrigues formulae in different forms. In this paper our attempt is to unify all such polynomials and to derive bilateral generating relations for the polynomial set $R_n^{(\alpha,\beta)}(x)$ defined by (1.1) below.

1. We define the polynomial set $R_n(x)$ by means of the Rodrigues formula

$$R_n^{(\alpha,\beta)}[x;\, a,\, b,\, c,\, d;\, p,\, q,\, \gamma,\, \xi;\, w(x)] \equiv R_n(x)$$

$$= \frac{(ax^p + b)^{-\alpha}\,(ex^q + d)^{-\beta}}{K_n\, w(x)}\, T^n_{k;\lambda}\Big[(ax^p + b)^{\alpha+\gamma n}\,(cx^q + d)^{\beta+\xi n}\, W(x)\Big],$$

$$n = 0, 1, 2, \ldots \qquad \ldots(1.1)$$

where

$$T_{k;\lambda} = x^k(\lambda + xD),\;\; D \equiv \frac{d}{dx} \qquad \ldots(1.2)$$

K_n is some constant and $w(x)$ is any general function of x, differentiable any number of times.

Some important and obvious connections of our polynomial set with those of the earlier workers are:

(1) with $p = q = 1$, $K_n = n!$, $\lambda = 0$, $w(x) = \exp(-px^r)$, $a = d = 1$, $b = c = 0$, $\gamma = -k$ and $\alpha = a + kn$ (1.1) reduces to the polynomial set $G_n^{(a)}(x, r, p, k)$, defined by Srivastava and Singhal (1971).

(2) $p = q = 1$, $K_n = n!$, $\lambda = 0$, $k = -1$, reduces the polynomial set $R_n(x)$ to $S_n^{(\alpha,\beta)}[x;\, a,\, b,\, c,\, d;\, \gamma,\, \xi;\, w(x)]$, defined by Srivastava and Panda (1975). Obviously, the polynomial set defined by Patil and Thakare (1977), too becomes a special case of (1.1).

Main results of the present paper consist of the generating relations:

$$\sum_{n=0}^{\infty} R_{n+m}^{(\alpha-\gamma n,\beta-\xi n)}\,[x;\,a,\,b,\,c,\,d;\,p,\,q,\,\gamma,\,\xi;\,w(x)]\,\frac{t^n}{n!}$$

$$= (1-ktX')^{-\lambda/k}\,\frac{w\{x(1-ktX')^{-1/k}\}}{w(x)}\left[\frac{ax^p(1-ktX')^{-p/k}+b}{ax^p+b}\right]^{\alpha}$$

$$\times\left[\frac{cx^q(1-ktX)^{-q/k}+d}{cx^q+d}\right]^{\beta} R_m^{(\alpha,\beta)}\,[x(1-ktX')^{-1/k};$$

$$a,\,b,\,c,\,d;\,p,\,q,\,\gamma,\,\xi;\,w(x(1-ktX')^{-1/k})] \quad \ldots(1.3)$$

where

$$X' = x^k(ax^p+b)^{\gamma}\,(cx^q+d)^{\xi} \quad \ldots(1.4)$$

and

$$\sum_{n=0}^{\infty} R_n^{(\alpha-\gamma n,\beta-\xi n)}\,[x;\,a,\,b,\,c,\,d;\,p,\,q,\,\gamma,\,\xi;\,w(x)]\,\Phi_n(y)t^n$$

$$= (1-ktX')^{-\lambda/k}\,\frac{w\{x(1-ktX)^{-1/k}\}}{w(x)}\left[\frac{ax^p\,(1-ktX')^{-p/k}+b}{ax^p+b}\right]^{\alpha}$$

$$\times\left[\frac{cx^q(1-ktX')^{-q/k}+d}{cx^q+d}\right]^{\beta} F[x(1-ktX')^{-1/k};\,X''] \quad \ldots(1.5)$$

where X' is given by (1.4),

$$X'' = yt\left[\frac{ax^p(1-ktX')^{-p/k}+b}{ax^p+b}\right]^{\gamma}\left[\frac{cx^q(1-ktX')^{-q/k}+d}{cx^q+d}\right]^{\xi}$$

$$F[x;\,t] = \sum_{n=0}^{\infty} \mu_n\,R_n^{(\alpha-\gamma n,\,\beta-\xi n)}\,[x;\,a,\,b,\,c,\,d;\,p,\,q,\,\gamma,\,\xi;\,w(x)]\,\frac{t^n}{n!} \quad \ldots(1.6)$$

$\Phi_n(y)$ is a polynomial of degree n in y given by

$$\Phi_n(y) = \frac{1}{n!}\sum_{j=0}^{n}\binom{n}{j}\mu_j y^j$$

and $\mu_j \neq 0$ are arbitrary constants.

§2. We now establish (1.3) from (1.1). We have

$$\sum_{n=0}^{\infty} R_{n+m}^{(\alpha-\gamma\overline{n+m},\;\beta-\xi\overline{n+m})}\,[x]\,\frac{t^n}{n!} =$$

(*equation continued on p.* 379)

$$= \frac{(ax^p + b)^{-\alpha+\gamma m}(cx^q + d)^{-\beta+\xi n}}{w(x)} \sum_{n=0}^{\infty} \frac{t^n}{n!} (ax^p + b)^{\gamma n} (cx^q + d)^{\xi n}$$

$$\times T^m_{k;\lambda} \{(ax^p + b)^\alpha (cx^q + a)^\beta w(x)\}].$$

Using (1.1) and a property of $T_{k;\lambda}$, viz.

$$\exp (tT_{k;\lambda}) f(x) = (1 - ktx^k)^{-\lambda/k} f(x(1 - ktx^k)^{-1/k})$$

and after a little calculation we get the required result (1.3).

§3. For the proof of bilateral generating relation (1.5), let us start with (cf. Singhal and Srivastava 1972, p. 756, section 2)

$$\sum_{n=0}^{\infty} R_n^{(\alpha-\gamma n, \beta-\xi n)} [x] \, \Phi_n(y) \, t^n$$

$$= \sum_{j=0}^{\infty} \mu_j \frac{(yt)^j}{j!} \sum_{n=0}^{\infty} \frac{t^n}{n!} R_{n+j}^{(\alpha-\gamma\overline{n+j}, \beta-\xi\overline{n+j})} [x].$$

Now the result (1.5) would follow, if we interpret the above expression by (1.3) and (1.6) (see also Srivastva and Panda 1975, p. 312, Theorem 1).

§4. *Applications* — Since the polynomial set $R_n(x)$ incorporates in itself several classical as well as other polynomials, a large variety of generating relations for the abovementioned polynomials may be obtained by assigning different values to the parameters in $R_n^{(\alpha,\beta)} (x)$.

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