

Converse theorems assuming a partial euler product

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Abstract

Associated to a newform $f(z)$ is a Dirichlet series $L_f(s)$ with functional equation and Euler product. Hecke showed that if the Dirichlet series $F(s)$ has a functional equation of a particular form, then $F(s) = L_f(s)$ for some holomorphic newform $f(z)$ on $\Gamma(1)$. Weil extended this result to $\Gamma_0(N)$ under an assumption on the twists of $F(s)$ by Dirichlet characters. Conrey and Farmer extended Hecke's result for certain small N , assuming that the local factors in the Euler product of $F(s)$ were of a special form. We make the same assumption on the Euler product and describe an approach to the converse theorem using certain additional assumptions. Some of the assumptions may be related to second order modular forms.

Keywords

Converse theorem Hecke operator L -function Modular form

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