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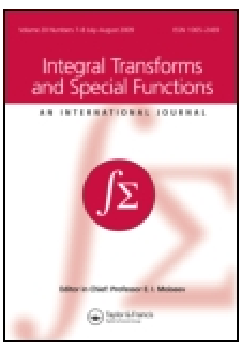

Euler–Frobenius numbers

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Wolfgang Gawronski[a] & Thorsten Neuschel[a*]

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Abstract

These numbers are defined as the coefficients of the Euler–Frobenius polynomials

$$P_{n,\lambda}(z) = \sum_{l=0}^{n} A_{n,l}(\lambda) z^l,$$

which usually are introduced via the rational function expansion

$$\sum_{\nu=0}^{\infty} (\nu + \lambda)^n z^\nu = \frac{P_{n,\lambda}(z)}{(1-z)^{n+1}},$$

n being a nonnegative integer and $\lambda \in [0, 1)$. The special case $A_{n,l}(0)$ is known from combinatorics (Eulerian numbers) and the general one $A_{n,l}(\lambda)$ occurs, for example, in approximation theory, summability, and rounding error analysis. By supplementing and extending known results on Eulerian numbers, various theorems for the Euler–Frobenius numbers $A_{n,l}(\lambda)$ and related quantities are established including unimodality, monotonicity properties, and asymptotic expansions given by a local central limit theorem.

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Keywords

- Eulerian numbers,
- Euler–Frobenius polynomials,
- local central limit expansions,
- rounding errors,
- 05D40,
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Euler–Frobenius numbers

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(*Final version received 21 December 2012*)

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Keywords: Eulerian numbers; Euler–Frobenius polynomials; local central limit expansions; rounding errors

2010 Mathematics Subject Classifications: 05D40; 41A40

1. Introduction and summary

In this article, we are concerned with the coefficients of the Euler–Frobenius polynomials $P_{n,\lambda}$, which can be generated from the geometric series through the representations

$$\sum_{\nu=0}^{\infty} (\nu+\lambda)^n z^\nu = \left(\lambda + z\frac{\mathrm{d}}{\mathrm{d}z}\right)^n \frac{1}{1-z} = \frac{P_{n,\lambda}(z)}{(1-z)^{n+1}}, \qquad (1.1)$$

n being a nonnegative integer and the parameter λ is considered to satisfy $\lambda \in [0,1)$, for example, [1, p. 7], Problem 46 in case $\lambda = 0$. For the power series, being convergent for $|z| < 1$, the two right-hand expressions may serve as analytic extensions onto the punctured complex plane $\mathbb{C}\setminus\{1\}$. This function is a special case of Lerch's transcendental function, cf. [2, 3, p. 33], which plays an important role in various parts of mathematics and related fields. For instance, it occurs in

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