

Central Delannoy Numbers and Balanced Cohen-Macaulay Complexes

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Dedicated to my father, Gábor Hetyei, on the occasion of his 70th birthday

Abstract. We introduce a new join operation on colored simplicial complexes that preserves the Cohen-Macaulay property. An example of this operation puts the connection between the central Delannoy numbers and Legendre polynomials in a wider context.

Keywords: balanced simplicial complex, Delannoy numbers, Cohen-Macaulay property

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