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Arithmetic properties of Apéry numbers

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Abstract

Let $(A_n)_{n\geq 1}$ be the sequence of Apéry numbers with a general term given by $A_n = \sum_{k=0}^{n} \binom{n}{k}^2 \binom{n+k}{k}^2$. In this paper, we prove that both the inequalities $\omega(A_n) > c_0 \log\log\log n$ and $P(A_n) > c_0 (\log n \log\log n)^{1/2}$ hold for a set of positive integers n of asymptotic density 1. Here, $\omega(m)$ is the number of distinct prime factors of m, $P(m)$ is the largest prime factor of m and $c_0 > 0$ is an absolute constant. The method applies to more general sequences satisfying both a linear recurrence of order 2 with polynomial coefficients and certain Lucas–type congruences.

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