PUBLICATIONS DE L'INSTITUT MATHÉMATIQUE
Nouvelle série, tome 23 (37) 1978, p.p. 169—177

CLASSICAL POLYNOMIALS — A UNIFIED PRESENTATION

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(Received June 26 1977)

1. Introduction:

The classical polynomials have a generalised Rodrigues' formula of the form

$$F_n(x)=\frac{1}{K_n\cdot\omega(x)}D^n[\omega(x)\cdot X^n], \tag{1.1}$$

where K_n is a constant, X is a function in x, whose coefficients are independent of n, and $w(x)$ is the weight function, where $F_n(x)$ is a polynomial in x.

Most familar polynomials defined in this manner are as follows:

$$P_n(x)=\frac{1}{2^n n!}D^n(x^2-1)^n \quad \text{— Legendre polynomials.} \tag{1.2}$$

(1.3.) $$P_n^{(\alpha,\beta)}(x)=\frac{(-1)^n}{2^n\cdot n!}(1-x)^{-\alpha}(1+x)^{-\beta}D^n[(1-x)^{\alpha+n}(1+x)^{\beta+n}]$$

— Jacobi polynomials.

$$C_n^{(\lambda)}(x)=\frac{(-1)^n}{2^n\cdot n!}(1-x^2)^{-\lambda+\frac{1}{2}}D^n\left[(1-x^2)^{\lambda-\frac{1}{2}+n}\right] \tag{1.4}$$

— Gegenbauer polynomials.

$$R_{2n}(x)=D^n[x^n(1-x^2)^n] \quad \text{— Appell [1].} \tag{1.5}$$

$$L_n^{(\alpha)}(x)=\frac{1}{n!}x^{-\alpha}e^x D^n[x^{\alpha+n}e^{-x}] \quad \text{— Laguerre polynomials.} \tag{1.6}$$

$$H_n(x)=(-1)^n e^{x^2}D^n(e^{-x^2}) \quad \text{— Hermite polynomials.} \tag{1.7}$$

$$y_n(x, a+2, b)=b^{-n}x^{-a}e^{b/x}D^n[x^{a+2n}\cdot e^{-b/x}] \tag{1.8}$$

— Bessel polynomials.

$$h_n(x)=\frac{1}{n!}e^{x^2}D^n(x^n e^{-x^2}) \quad \text{— Humbert polynomials.} \tag{1.9}$$

Generalisation of Rodrigues' formula has been a starting point of many researches in the past. And in this direction Menon [8] generalised Legendre polynomials by defining the polynomials

(1.10) $$P_{n,s}(z)=\frac{1}{n!\,s^n}D^n(z^s-1)^n.$$

Gould-Hoper [7], Chatterjea [2, 3], Singh-Srivastava [12] generalised Hermite polynomials, Laguerre polynomials, Humbert polynomials etc. by the following relations:

(1.11) $$H_n^r(x, a, p)=(-1)^n x^{-a}e^{px^r}D^n(x^a e^{-px^r}),$$

(1.12) $$L_n^\alpha(x, r, p)=T_{rn}^{(\alpha)}(x, p)=\frac{x^{-\alpha}}{n!}e^{px^r}D^n(x^{a+n}e^{-px^r}),$$

(1.13) $$F_n^{(r)}(x, \alpha, m, p)=x^{-\alpha}e^{px^r}D^n[x^{\alpha+mn}e^{-px^r}].$$

Also there have been attempts by Fuziwara [6], Chatterjea [5] and many others to give a Rodrigues' formula to include all familiar classical polynomials, and they succeeded to some extent. In this direction it is of interest to study a set of polynomials $P_n^{(\alpha,\beta,k)}(x, r, s, m)$, defined by relation

(1.14) $$P_n^{(\alpha,\beta,k)}(x, r, s, m)=x^{-\alpha}(1-kx^r)^{-\beta/k}D^n[x^{\alpha+mn}(1-kx^r)^{\beta/k+sn}],$$

where α, β, k, r, s and m are parameters.

This function happens to include all the polynomials and functions given above from (1.2) to (1.13). Following are particular cases of (1.14);

(1.15) $$P_n(x)=(-1)^n\frac{2^{-n}}{n!}P_n^{(0,0,1)}(x, 2, 1, 0)=\frac{(-1)^n}{n!}P_n^{(0,0,1)}\left(\frac{1+x}{2}, 1, 1, 1\right)$$
$$=\frac{1}{n!}P_n^{(0,0,1)}\left(\frac{1-x}{2}, 1, 1, 1\right).$$

(1.16) $$P_n^{(\alpha,\beta)}(x)=\frac{(-1)^n}{n!}P_n^{(\alpha,\beta,1)}\left(\frac{1+x}{2}, 1, 1, 1\right)=\frac{1}{n!}P_n^{(\beta,\alpha,-1)}\left(\frac{1-x}{2}, 1, 1, 1\right).$$

(1.17) $$C_n^{(\lambda)}(x)=\frac{(-1)^n}{n!}P_n^{\left(\lambda-\frac{1}{2},\lambda-\frac{1}{2},1\right)}\left(\frac{1+x}{2}1, 1, 1\right)$$
$$=\frac{1}{n!}P_n^{\left(\lambda-\frac{1}{2},\lambda-\frac{1}{2},1\right)}\left(\frac{1-x}{2}, 1, 1, 1\right)=\frac{(-1)^n}{2^n\cdot n!}P_n^{\left(0,\lambda-\frac{1}{2},1\right)}(x, 2, 1, 0).$$

(1.18) $$R_{2n}(x)=P_n^{(0,0,1)}(x, 2, 1, 1).$$

(1.19) $$L_n^{(\alpha)}(x)=\lim_{K\to 0}\frac{1}{n!}P_n^{(\alpha,1,k)}(x, 1, 0, 1).$$

(1.20) $$H_n(x)=\lim_{K\to 0}(-1)^n P_n^{(0,2,k)}(x, 1, 0, 0)=\lim_{K\to 0}(-1)^n P_n^{(0,1,k)}(x, 2, 0, 0)$$

(1.21) $$y_n(x, a+2, b) = \lim_{K\to 0} b^{-n} P_n^{(a,b,k)}(x, -1, 0, 2).$$

(1.22) $$h_n(x) = \lim_{K\to 0} \frac{1}{n!} P_n^{(0,1,K)}(x, 2, 0, 1) = \lim_{K\to 0} \frac{1}{n!} P_n^{(0,2,K)}(x, 1, 0, 1).$$

(1.23) $$P_{n,s}(x) = \frac{(-1)^n}{n!\, s^n} P_n^{(0,0,1)}(x, s, 1, 0).$$

(1.24) $$H_n^r(x, a, p) = \lim_{K\to 0} (-1)^n P_n^{(a,p,K)}(x, r, 0, 0).$$

(1.25) $$T_{rn}^{(\alpha)}(x, p) = L_n^{(\alpha)}(x, r, p) = \lim_{K\to 0} \frac{1}{n!} P_n^{(\alpha,p,K)}(x, r, 0, 1).$$

(1.26) $$F_n^{(r)}(x, a, m, p) = \lim_{K\to 0} P_n^{(\alpha,p,K)}(x, r, 0, m).$$

2. Expansion and generating function

From (1.14), it is easily seen that explicit form for $P_n^{(\alpha,\beta,k)}(x, r, s, m)$ is

(2.1) $$P_n^{(\alpha,\beta,k)}(x, r, s, m) = n!\, x^{(m-1)n} (1-kx^r)^{sn} \cdot \sum_{i=0}^{n} (-1)^i \frac{(-\beta-ksn)^{(k,i)}}{i!} \left(\frac{x^r}{1-kx^r}\right)^i \sum_{t=0}^{i} \binom{i}{t} \binom{\alpha+mn+rt}{n}$$

where $(a)^{(k,i)} = a(a+k)(a+2k)\cdots(a+(i-1)k)$.

Also from (1.14) we have

(2.2) $$\sum_{n=0}^{\infty} \frac{t^n}{n!} P_n^{(\alpha,\beta,k)}(x, r, s, m) =$$
$$= x^{-\alpha}(1-kx^r)^{-\beta/k} \sum_{n=0}^{\infty} \frac{t^n}{n!} D^n [\{x^m(1-kx^r)^s\}^n x^\alpha (1-kx^r)^{\beta/k}],$$

and interpreting the last expression with the help of Langrange's theorem [9],

(2.3) $$\frac{f(y)}{1-t\Phi'(y)} = \sum_{n=0}^{\infty} \frac{t^n}{n!} D^n \{[\Phi(x)]^n f(x)\}, \text{ where}$$

$y = x + t\Phi(y)$, we find that

(2.4) $$\sum_{n=0}^{\infty} \frac{t^n}{n!} P_n^{(\alpha,\beta,k)}(x, r, s, m)$$
$$= \left(\frac{z}{x}\right)^\alpha \left(\frac{1-kz^r}{1-kx^r}\right)^{\beta/k} \{1 - tz^{m-1}(1-kz^r)^{s-1} \cdot (m - kz^r(m+sr))\}^{-1}$$

where $z = x + tz^m(1-kz^r)^s$.

This gives a generating function for (1.14). In view of the relations (1.15) to (1.26), (2.4) yields a large number of generating functions for the polynomials of Jacobi, Hermite, Laguerre, Bessel, Gegenbauer, etc.. For instance we have the special cases:

Legendre polynomials —

$$\sum_{n=0}^{\infty} (2t)^n P_n(x) = (1+2tz)^{-1},$$

where

$$z = x + t(1-z^2),$$

which on simplification gives [10]

$$\sum_{n=0}^{\infty} u^n P_n(x) = (1-2ux+u^2)^{-1/2}. \tag{2.5}$$

Jacobi polynomials [10] —

$$\sum_{n=0}^{\infty} P_n^{(\alpha,\beta)}(x)\, t^n = 2^{\alpha+\beta} R^{-1} (1-t+R)^{-\alpha} (1+t+R)^{-\beta}, \tag{2.6}$$

where

$$R = (1-2xt+t^2)^{1/2}.$$

Generalised Hermite function [7] –

$$\sum_{n=0}^{\infty} \frac{t^n}{n!} H_n^{(r)}(x, a, p) = x^{-\alpha} (x-t)^{\alpha} e^{p(x^r-(x-t)^r)}. \tag{2.7}$$

Generalised Laguerre function [2, 12] —

$$\sum_{n=0}^{\infty} t^n L_n^{(\alpha)}(x, r, p) = (1-t)^{-\alpha-1} e^{px^r(1-(1-t)^{-r})}. \tag{2.8}$$

Chatterjea's generalised function [3.4] –

$$\sum_{n=0}^{\infty} \frac{t^n}{n!} F_n^{(r)}(x, a, m, p) = \left(\frac{z}{x}\right)^{\alpha} (1-mtz^{m-1})^{-1} e^{p(x^r-z^r)}, \tag{2.9}$$

where

$$z = x + tz^m.$$

Now using the Taylor's expansion

$$f(x+tx^k) = \sum_{n=0}^{\infty} \frac{t^n x^{kn}}{n!} D^n f(x),$$

we get another generating function as

$$\sum_{n=0}^{\infty} \frac{t^n}{n!} P_n^{(\alpha-mn,\, \beta-ksn,\, k)}(x, r, s, m) = (1+tx^{m-1}(1-kx^r)^s)^{\alpha} \cdot \tag{2.10}$$

$$\cdot (1-kx^r)^{-\beta/k} \{1-kx^r(1+tx^{m-1}(1-kx^r)^s)^r\}^{\beta/k}.$$

Also we have

(2.11)
$$\sum_{n=0}^{\infty} \frac{t^n}{n!} P_n^{(\alpha-mn+n,\, \beta-ksn+kn,\, k)}(x, r, s, m) =$$
$$= \left(\frac{z}{x}\right)^{\alpha} \left(\frac{1-kz^r}{1-kx^r}\right)^{\beta/k} \{1 - tx^{m-1}(1-kx^r)^{s-1} \cdot (1-k(r+1)z^r)\}^{-1},$$

where
$$z = x + tzx^{m-1}(1-kz^r)(1-kx^r)^{s-1},$$

(2.12)
$$\sum_{n=0}^{\infty} P_n^{(\alpha,\, \beta-ksn,\, k)}(x, r, s, m) \frac{t^n}{n!} = \left(\frac{z}{x}\right)^{\alpha} \left(\frac{1-kz^r}{1-kx^r}\right)^{\beta/k} \cdot$$
$$\cdot \{1 - mt(1-kx^r)^s z^{m-1}\}^{-1},$$

where
$$z = x + tz^m y \cdot (1-kx^r)^S$$

and

(2.13)
$$\sum_{n=0}^{\infty} \frac{t^n}{n!} P_n^{(\alpha-mn+n,\, \beta-ksn,\, k)}(x, r, s, m) \left(\frac{1}{x^{m-1}(1-kx^r)^S}\right)^n$$
$$= (1-t)^{-\alpha-1} \left(\frac{1-kx^r(1-t)^{-r}}{1-kx^r}\right)^{\beta/k}.$$

These relations also yield into interesting generating relations for the familiar classical polynomials.

3. Orthogonality type relations:

From (2.1) we observe that $P_n^{(\alpha,\beta,k)}(x, r, s, m)$ is a polynomial of degree $(m+rs-1)n$ when $k \neq 0$, and of degree $(m+r-1)n$ when $k=0$.

If r is not even integer and $k \neq 0$, $\eta(l+pq-1)$ is positive integer, then it is easily seen that

(3.1)
$$\int_0^{(1/k)^{1/r}} x^{\alpha}(1-kx^r)^{\beta/k} P_{\mu}^{(\alpha,\beta,k)}(x, r, s, m) \cdot P_{\eta}^{(\gamma,\delta,k)}(x, p, q, l) =$$
$$= 0;\ \mu > \eta(l+pq-1),$$
$$\neq 0;\ \mu = \eta(l+pq-1),$$

and when $k=0$ and $\eta(l+p-1)$ is a positive integer

(3.2)
$$\int_0^{\infty} x^{\alpha} e^{-\beta x^r} P_{\mu}^{(\alpha,\beta,0)}(x, r, 0, m)\, P_{\eta}^{(\gamma,\delta,0)}(x, p, 0, l) = 0;\ \mu > \eta(l+p-1)$$
$$\neq 0,\ \mu = \eta(l+p-1).$$

In the case r be an even integer, the polynomials admit the following integrals only in case $r=2$, and when $k \neq 0$ and $\eta(l+pq-1)$ is a positive integer

$$(3.3) \qquad \int_{-\frac{1}{\sqrt{k}}}^{\frac{1}{\sqrt{k}}} x^{\alpha}(1-kx^2)^{\beta/k} P_{\mu}^{(\alpha,\beta,k)}(x, 2, s, m)\, P_{\eta}^{(\gamma,\delta,k)}(x, 2, q, l)\, dx$$

$$= 0; \; \mu > \eta(l+pq-1),$$

$$\neq 0; \; \mu = \eta(l+pq-1),$$

and when $k=0$, $\eta(l+p-1)$ is a positive integrer

$$(3.4) \qquad \int_{-\infty}^{\infty} x^{\alpha} e^{-\beta x^2} P_{\mu}^{(\alpha,\beta,0)}(x, 2, 0, m)\, P_{\eta}^{(\gamma,\delta,0)}(x, 2, 0, l)\, dx$$

$$= 0; \; \mu > \eta(l+p-1),$$

$$\neq 0; \; \mu = \eta(l+p-1).$$

The values of the above integrals are easily obtainable for $\mu=\eta(l+pq-1)$ or $\mu=\eta(l+p-1)$, as the case may be, by application of successive integration by parts. Since these values are not required here for any use, hence have not been obtained. It is also easily seen that, in particular cases, above integrals hold.

4. Recurrence and other relations:

Following recurrence relations are easily seen to satisfy by

$$P_n^{(\alpha,\beta,k)}(x, r, s, m):$$

$$(4.1) \qquad DP_n^{(\alpha,\beta,k)}(x, r, s, m) + \left(\frac{\alpha}{x} - r\beta x^{r-1}(1-kx^r)^{-1}\right) P_n^{(\alpha,\beta,k)}(x, r, s, m)$$

$$= P_{n+1}^{(\alpha-m,\beta-sk,k)}(x. r, s, m) \cdot x^{-m}(1-kx^r)^{-s},$$

and

$$(4.2) \qquad P_{n+1}^{(\alpha,\beta,k)}(x, r, s, m) = (\alpha+nm+m)\, x^{m-1}(1-kx^r)^s P_n^{(\alpha+m-1,\beta+ks,k)}(x, r, s, m)$$

$$- r(\beta+k(n+1)s)\, x^{m+r-1}(1-kx^r)^{s-1} P_n^{(\alpha+m-1,\beta+ks-k,k)}(x, r, s, m).$$

Interestingly (4.1) leads to

$$\left(D+\frac{\alpha}{x} - r\beta\frac{x^{r-1}}{1-kx^r}\right) P_n^{(\alpha,\beta,k)}(x, r, s, m) = x^{-m}(1-kx^r)^{-s} P_{n+1}^{(\alpha-m,\beta-sk,k)}(x, r, s, m).$$

Letting $\mathfrak{D} = D + \frac{\alpha}{x} - \frac{r\beta x^{r-1}}{1-kx^r}$, we have

$$(4.3) \qquad \mathfrak{D} P_n^{(\alpha,\beta,k)}(x, r, s, m) = x^{-m}(1-kx^r)^{-s} P_{n+1}^{(\alpha-m,\beta-sk,k)}(x, r, s, m),$$

and repeated operations of $\mathfrak{D}$ give

$$(4.4)\qquad \mathfrak{D}^t P_n^{(\alpha,\beta,k)}(x, r, s, m) = x^{-tm}(1-kx^r)^{-st} P_{n+t}^{(\alpha-tm,\,\beta-skt,\,k)}(x, r, s, m).$$

$\mathfrak{D}$ is the operator of Gould-Hopper [7] for $k=0$ and hence obviously gives, as particular cases the following [7, 11]:

$$(4.5)\qquad \mathfrak{D}^t H_n^r(x, \alpha, \beta) = (-1)^t H_{n+t}^r(x, \alpha, \beta).$$

$$(4.6)\qquad \mathfrak{D}^t T_{rn}^{(\alpha)}(x, \beta) = \binom{n+t}{n} t!\, x^t\, T_{r(t+n)}^{(\alpha-t)}(x, \beta).$$

$$(4.7)\qquad \mathfrak{D}^t \beta^n y_n(x, \alpha+2, \beta) = x^{-2t} y_{n+t}(x, \alpha+2-2m, \beta),$$

$$(4.8)\qquad \mathfrak{D}^t F_n^{(r)}(x, \alpha, m, \beta) = x^{-mt} F_{n+t}^{(r)}(x, \alpha-kt, k, \beta).$$

In case of Jacobi polynomials we have

$$(4.9)\qquad \left(D+\frac{\alpha}{x+1}+\frac{\beta}{x-1}\right)^t P_n^{(\alpha,\beta)}(x) = \binom{n+t}{n} t!\, 2^{-n-2t}(x^2-1)^{-t} P_{n+t}^{(\alpha-t,\,\beta-t)}(x).$$

It is easily seen that

$$(4.10)\qquad \mathfrak{D}^n(U\cdot V) = \sum_{i=0}^{n} \binom{n}{i} \mathfrak{D}^{n-i} U \cdot D^i V.$$

This relation is analogous to that of Gould-Hopper [7]. Using (4.4) for $n=0$, we get

$$(4.11)\qquad \mathfrak{D}^n \equiv \sum_{i=0}^{n} \binom{n}{i} x^{-m(n-i)}(1-kx^r)^{-s(n-i)} P_{n-i}^{(\alpha-m(n-i),\,\beta-sk(n-i),\,k)}(x, r, s, m)\cdot D^i.$$

Again we see that

$$D^j P_n^{(\alpha,\beta,k)}(x, r, s, m) = \sum_{i=0}^{j} \binom{j}{i} P_{j-i}^{(-\alpha,\,-\beta,\,k)}(x, r, 0, 0)\, \mathfrak{D}^i P_n^{(\alpha,\beta,k)}(x, r, s, m).$$

This suggests inverse relation to (4.11) as

$$(4.12)\qquad D^j = \sum_{i=0}^{j} \binom{j}{i} P_{j-i}^{(-\alpha,\,-\beta,\,k)}(x, r, 0, 0)\, \mathfrak{D}^i.$$

Suppose that $f(x+t)$ possesses a power series in powers of t as

$$f(x+t) = \sum_{n=0}^{\infty} \frac{t^n}{n!} D^n f(x).$$

Then using (2.9) and (4.11) and simplifying we get

$$(4.13)\qquad e^{t\mathfrak{D}}\cdot f(x) = \sum_{j=0}^{\infty} \frac{t^j}{j!} \mathfrak{D}^j f(x)$$

$$= \left(1+\frac{t}{x}\right)^{\alpha}(1-kx^r)^{-\beta/k}\left\{1-kx^r\left(1+\frac{t}{x}\right)^r\right\}^{\beta/k}\cdot f(x+t).$$

Hence choosing $f(x)=P_n^{(\alpha,\beta,k)}(x, r, s, m)$ and using (4.4) we obtain

$$(4.14)\qquad \sum_{j=0}^{\infty}\frac{t^j}{j!}P_{j+n}^{(\alpha-jm,\beta-jks,k)}(x, r, s, m)=(1+tx^{m-1}(1-kx^r)^s)^{\alpha}\cdot$$
$$\cdot(1-kx^r)^{-\beta/k}\{1-kx^r(1+tx^{m-1}(1-kx^r)^s)^r\}^{\beta/k}\cdot$$
$$\cdot P_n^{(\alpha,\beta,k)}(x+tx^{m-1}(1-kx^r)^s, r, s, m).$$

Also we get by use of (2.12)

$$(4.15)\qquad \sum_{n=0}^{\infty}\frac{1}{n!}P_{n+u}^{(\alpha-mn+n,\beta-ksn,k)}(x, r, s, m)\cdot\left(\frac{t}{x^{m-1}(1-kx^r)^s}\right)^n$$
$$=(1-t)^{-u-1-\alpha}\left(\frac{1-kz^r}{1-kx^r}\right)^{\beta/k}P_u^{(\alpha,\beta,k)}(z, r, s, m),$$

where

$$z=x/(1-t).$$

The generating relations (4.14) and (4.15) reduce to (2.9) and (2.12) respectively for $n=0$. These ıelations are of particular interest as we can use them to obtain bilateral generating functions.

5. Bilateral generating functions:

In this section we apply the generating function (4.14) and (4.15) to prove the following theorems:

Theorem 1. — *If*

$$(5.1)\qquad F(x, t)=\sum_{n=0}^{\infty}a_n P_n^{(\alpha-nm,\ \beta-ksn,k)}(x, r, s, m)\cdot\frac{t^n}{n!},$$

where a_n are arbitrary constants, then

$$(5.2)\qquad (1-tx^{m-1}(1-kx^r)^s)^{\alpha}(1-kx^r)^{-\beta/k}\{1-kx^r(1+tx^{m-1}(1-kx^r)^s\}^{\beta/k}\cdot$$
$$\cdot F\left[x+tx^{m-1}(1-kx^r)^s, \frac{yt}{(1-tx^{m-1}(1-kx^r)^s)^m(1-kx^r)^s\{1-kx^r(1+tx^{m-1}\cdot\cdot(1-kx^r)^s)^r\}^s}\right]=$$
$$=\sum_{n=0}^{\infty}P_n^{(\alpha-nm,\beta-ksn,k)}(x, r, s, m)\,\sigma_n(y)\cdot\frac{t^n}{n!},$$

where

$$(5.3)\qquad \sigma_n(y)=\sum_{u=0}^{n}\binom{n}{u}a_u y^u.$$

To prove this, substitute series expansion (5.3) of $\sigma_n(y)$ on *RHS* of (5.2) and we get

$$\sum_{u=0}^{\infty}a_u y^u\frac{t^u}{u!}\sum_{n=0}^{\infty}P_{n+u}^{(\alpha-(n+u),m,\ \beta-(n+u)sk,k)}(x, r, s, m)\frac{t^n}{n!}.$$

On summing the inner series with the help of (4.14) and then interpreting the expression with the help of (5.1), we get the theorem immediately.

Theorem 2. — *If*

$$(5.4)\quad G[x, t] = \sum_{n=0}^{\infty} a_n \frac{t^n}{n!} P_n^{(\alpha-mn+n,\,\beta-ksn,\,n)}(x, r, s, m)\cdot\left(\frac{1}{x^{m-1}(1-kx^r)^s}\right)^n$$

where a_n *are arbitrary constants, then*

$$(5.5)\quad (1-t)^{-1-\alpha}\left(\frac{1-kx^r(1-t)^{-r}}{1-kx^r}\right)^{\beta/k} G\left[\frac{x}{1-t}, \frac{yt}{x^{m-1}(1-t)^{m-2}(1-kx^r(1-t)^{-r})^s}\right]$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} P_n^{(\alpha-mn+n,\,\beta-ksn,\,n)}(x, r, s, m)\,\sigma_n(y)\left(\frac{t}{x^{m-1}(1-kx^r)^s}\right)^n,$$

where $\sigma_n(y)$ *are given by* (5.3)

Proof of this theorem is similar to that of theorem 1.

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