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CONGRUENCES ON THE L-FUNCTION OF AN ELLIPTIC CURVE PARAMETRISED BY MODULAR FUNCTIONS

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1. *Introduction*

In a previous paper [4], we showed how to determine the L-function of an elliptic curve of the form

$$E(D)\colon Dy^2z = x^3+Axz^2+Bz^3$$

where E is the curve

$$E\colon y^2z = x^3+Axz^2+Bz^3$$

of conductor N, and D is a square-free integer. Our determination used only a knowledge of the homology of $H/\Gamma^\circ(N)$ and the eigencycles therein corresponding to the curve E. The work was illustrated by the verification, in part, of the Birch–Swinnerton-Dyer conjectures (for which see, for instance, [5]) for $E_{17}(D)$, with small square-free D, where E_{17} is the curve parametrised by modular functions invariant under $\Gamma^\circ(17)$.

In [1], Manin makes empirical observations concerning congruence properties of the reflexive part of certain homology elements in $H/\Gamma^\circ(N)$, for $n = 11$, 17, 19 and 27.

It is not difficult to prove these and to extend them to other cases and this has been done by several others (Birch, Mazur, Stephens, all unpublished). A very natural proof of the general phenomenon is to be found in [2]. Unfortunately, the case of E_{17}, which we require, is not covered. In §2 we give a proof for the case E_{17} which will generalise easily to cover all cases for which such properties hold. We give a table for curves of conductor less than 100 showing the precise behaviour.

In §3, we obtain, as corollaries of Manin's congruences, congruences on N_p, the number of points on the reduction of E_{17} mod p. We also obtain somewhat stronger congruences by more direct methods. In §4, we use the results of §2 and §3 to prove parts of the Birch–Swinnerton-Dyer conjectures for $E_{17}(D)$ in several sequences of cases. The argument is related to that of Razar [3].

2. *Congruences on the homology elements*

In this, and in subsequent sections, we use freely the results and notation of [4]. We begin by recalling [4; equations (7) and (12)]

$$L_E{}^*(1, \chi) M_\infty(E) \prod_{p \in S \setminus \{\infty\}} n_p(E) = L_E(1, \chi) = \sum_{0 \leq b < d} \lambda_b I(b/d) \tag{1}$$

where χ has conductor d, with $(d, N) = 1$, and

$$\lambda_b = d^{-1} \sum_{(a,\, d)=1} e(-ab/d)\, \chi(a)$$

$$I(b/d) = -2\pi i \int_{b/d}^{i\infty} f(z)\, dz$$

where $f(z)\, dz$ is the differential on $H/\Gamma_0(N)$ corresponding to the elliptic curve E.

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