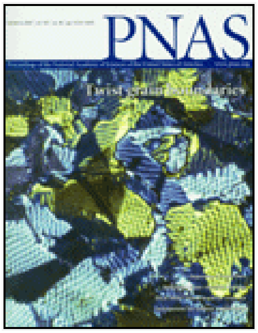

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*SOME BILINEAR GENERATING FUNCTIONS**

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Abstract.—In the present paper, the author applies some of his earlier results which extend the well-known Hille-Hardy formula for the Laguerre polynomials to certain classes of generalized hypergeometric polynomials in order to derive various generalizations of a bilinear generating function for the Jacobi polynomials proved recently by Carlitz. The corresponding results for the polynomials of Legendre, Gegenbauer (or ultraspherical), Laguerre, etc., can be obtained fairly easily as the specialized or limiting cases of the generating functions presented here. It is also shown how the formula of Carlitz follows rather rapidly from a result of Weisner involving the Gaussian hypergeometric functions.

1. *Introduction.*—For the Jacobi polynomial denoted by Szegö (ref. 5, p. 67) by

$$P_n^{(\alpha,\beta)}(x) = \sum_{k=0}^{n} \binom{n+\alpha}{k}\binom{n+\beta}{n-k}\left(\frac{x-1}{2}\right)^{n-k}\left(\frac{x+1}{2}\right)^{k}, \qquad (1)$$

Carlitz[1] has proved the bilinear generating function

$$\sum_{n=0}^{\infty} \frac{n!}{(\gamma)_n}(x-1)^n(y-1)^n P_n^{(\alpha-n,-\alpha-\gamma-n)}\left(\frac{x+1}{x-1}\right) P_n^{(\beta-n,-\beta-\gamma-n)}\left(\frac{y+1}{y-1}\right)t^n$$
$$= (1-t)^{-\alpha-\beta-\gamma}(1-xt)^{\alpha}(1-yt)^{\beta}\ {}_2F_1\left[\begin{matrix}-\alpha,-\beta; \\ \gamma;\end{matrix}\ \frac{(x-1)(y-1)t}{(1-xt)(1-yt)}\right], \qquad (2)$$

where

$$ {}_2F_1\left[\begin{matrix} a, b; \\ c; \end{matrix} z\right] = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}. \tag{3} $$

In particular, if $\gamma = -\alpha - \beta$, (2) would reduce to the elegant form (ref. 1, p. 89)

$$ \sum_{n=0}^{\infty} \frac{n!}{(-\alpha-\beta)_n} (x-1)^n (y-1)^n P_n^{(\alpha-n,\beta-n)}\left(\frac{x+1}{x-1}\right) P_n^{(\beta-n,\alpha-n)}\left(\frac{y+1}{y-1}\right) t^n $$
$$ = (1-xt)^{\alpha} (1-yt)^{\beta} {}_2F_1\left[\begin{matrix} -\alpha, -\beta; \\ -\alpha-\beta; \end{matrix} \frac{(x-1)(y-1)t}{(1-xt)(1-yt)}\right]. \tag{4} $$

In the present note it may be of interest to first show how the formula (2) follows rather rapidly from a well-known generating function due to Weisner, viz. (see ref. 6, p. 1037)

$$ \sum_{n=0}^{\infty} \frac{(\gamma)_n}{n!} {}_2F_1\left[\begin{matrix} -n, \alpha; \\ \gamma; \end{matrix} u\right] {}_2F_1\left[\begin{matrix} -n, \beta; \\ \gamma; \end{matrix} v\right] t^n $$
$$ = (1-t)^{\alpha+\beta-\gamma} [1+(u-1)t]^{-\alpha} [1+(v-1)t]^{-\beta} {}_2F_1\left[\begin{matrix} \alpha, \beta; \\ \gamma; \end{matrix} \zeta\right], \tag{5} $$

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