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# Some generalized Lagrange-based Apostol-Bernoulli, Apostol-Euler and Apostol-Genocchi polynomials

## Abstract

In this paper, we introduce a general family of Lagrange-based Apostol-type polynomials thereby unifying the Lagrange-based Apostol-Bernoulli and the Lagrange-based Apostol-Genocchi polynomials. We also define Lagrange-based Apostol-Euler polynomials via the generating function. In terms of these generalizations, we find new and useful relations between the unified family and the Apostol-Euler polynomials. We also derive their explicit representations and list some basic properties of each of them. Further relations between the above-mentioned polynomials, including a family of bilinear and bilateral generating functions, are given. Moreover, a generating relation involving the Stirling numbers of the second kind is derived.



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